

# Non-Deterministic Finite Automata dengan $\epsilon$ -Move

## Pertemuan 4

Mahasiswa mampu menggunakan dan menerapkan tahapan mesin Non-Deterministic Finite Automata dengan  $\epsilon$ -Move

# MATERI

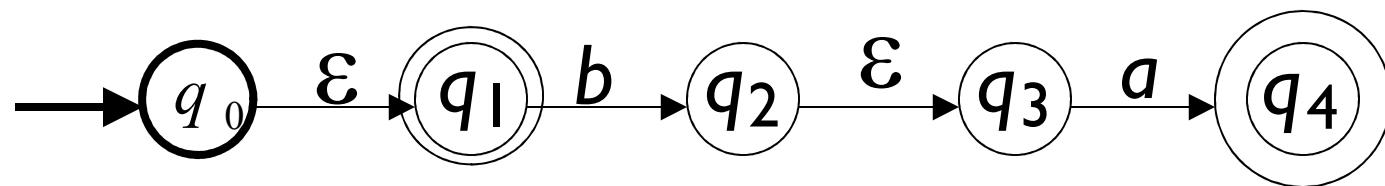
- Non-Deterministic Finite Automata dengan  $\epsilon$ -Move
- $\epsilon$ -Closure untuk suatu Non-Deterministic Finite Automata dengan  $\epsilon$ -Move
- Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

# Non Deterministic Finite Automata dengan $\epsilon$ -move

- NFA dengan  $\epsilon$ -move (transisi  $\epsilon$ ), diperbolehkan merubah state tanpa membaca input.
- Disebut dengan  $\epsilon$ -move karena tidak bergantung pada suatu input ketika melakukan transisi.
- Kegunaan  $\epsilon$ -move adalah untuk memudahkan mengkombinasikan finite state automata.

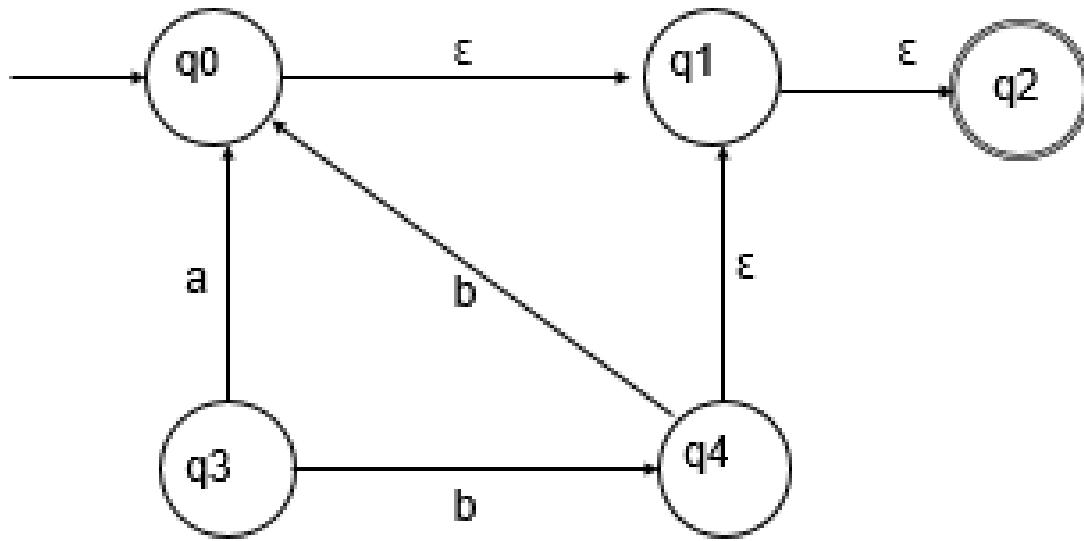
CONTOH

## NFA dengan $\epsilon$ - Move



Penjelasan : dari  $q_2$  tanpa membaca input dapat berpindah ke  $q_3$

# NFA dengan $\epsilon$ - Move



Dari  $q_0$  tanpa membaca input dapat berpindah ke  $q_1$

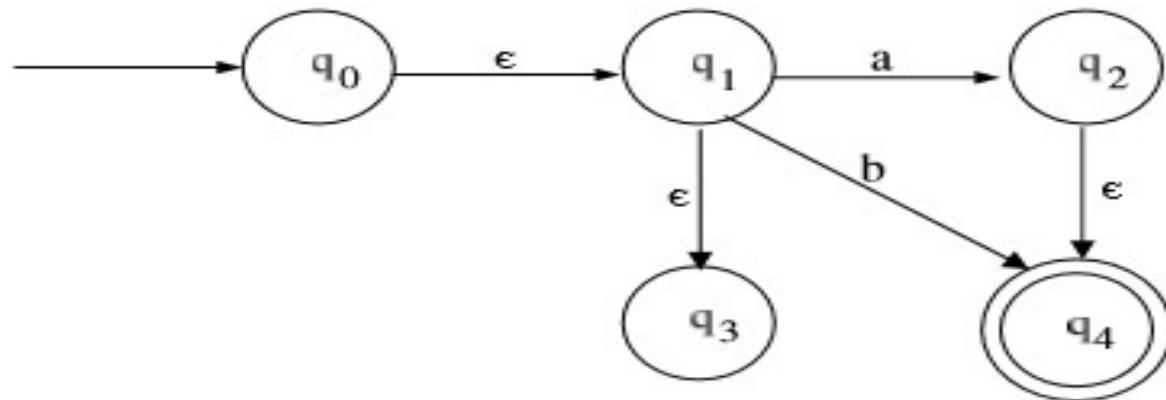
Dari  $q_1$  tanpa membaca input dapat berpindah ke  $q_2$

Dari  $q_4$  tanpa membaca input dapat berpindah ke  $q_1$

## **$\epsilon$ -closure untuk suatu NFA dengan $\epsilon$ -move**

- $\epsilon$ -closure adalah himpunan state-state yang dapat dicapai dari suatu state tanpa membaca input.
- $\epsilon$ -closure ( $q_0$ )=himpunan state-state yang dapat dicapai dari state  $q_0$  tanpa membaca input.
- Pada suatu state yang tidak memiliki  $\epsilon$ -move, maka  $\epsilon$ -closure nya adalah state itu sendiri.

# $\epsilon$ -Closure ( $\epsilon$ -Cl) untuk suatu Non-Deterministic Finite Automata dengan $\epsilon$ -Move



Dari gambar di atas, kita ketahui  $\epsilon$  – Closure untuk setiap state adalah sebagai berikut.

$$\epsilon \text{ – Closure} (q_0) = \{ q_0, q_1, q_3 \}$$

$$\epsilon \text{ – Closure} (q_1) = \{ q_1, q_3 \}$$

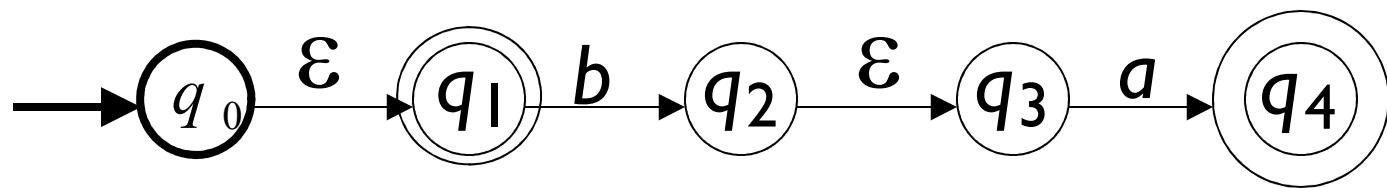
$$\epsilon \text{ – Closure} (q_2) = \{ q_2, q_4 \}$$

$$\epsilon \text{ – Closure} (q_3) = \{ q_3 \}$$

$$\epsilon \text{ – Closure} (q_4) = \{ q_4 \}$$

# $\varepsilon$ -Closure ( $\varepsilon$ -Cl) untuk suatu Non-Deterministic Finite Automata dengan $\varepsilon$ -Move

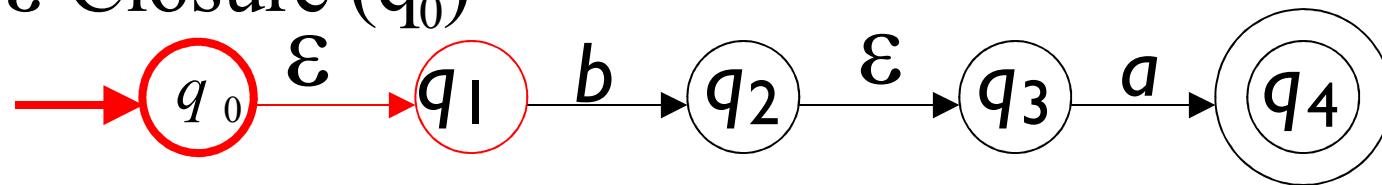
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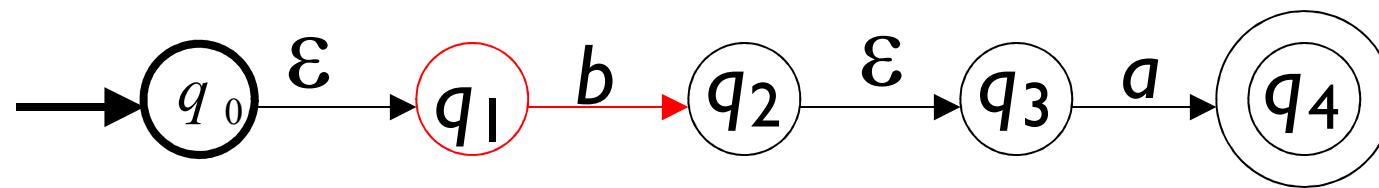
$\varepsilon$ -Closure dari gambar diatas ?

$\varepsilon$ -Closure  $\{q_0, q_1\}$  untuk setiap state

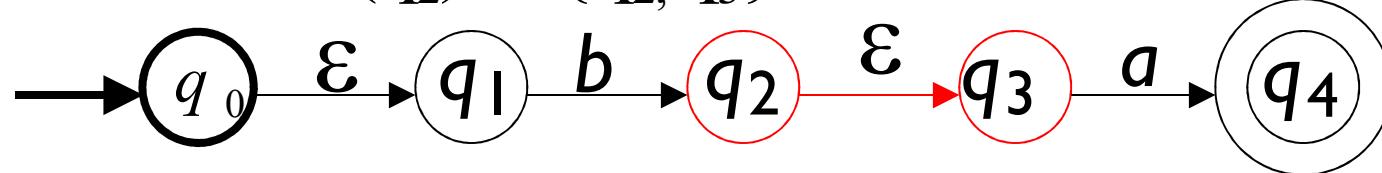
$\varepsilon$ -Closure ( $q_0$ ) =



$\varepsilon$ -Closure ( $q_1$ ) = {  $q_1$  }

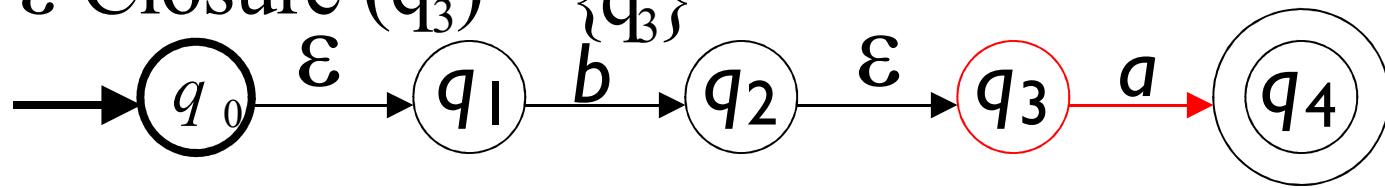


$\varepsilon$ -Closure ( $q_2$ ) = {  $q_2, q_3$  }

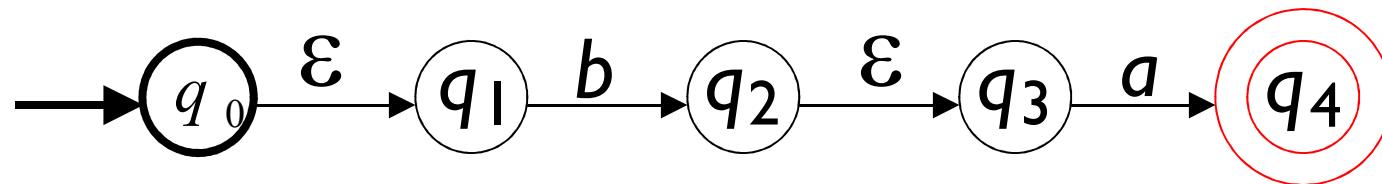


## $\epsilon$ -Closure untuk setiap state

$\epsilon$ -Closure ( $q_3$ ) =  $\{q_3\}$



$\epsilon$ -Closure ( $q_4$ ) =  $\{q_4\}$



## Ekuivalensi Non-Deterministic Finite Automata dengan $\epsilon$ -move ke Deterministic Finite Automata tanpa $\epsilon$ -Move

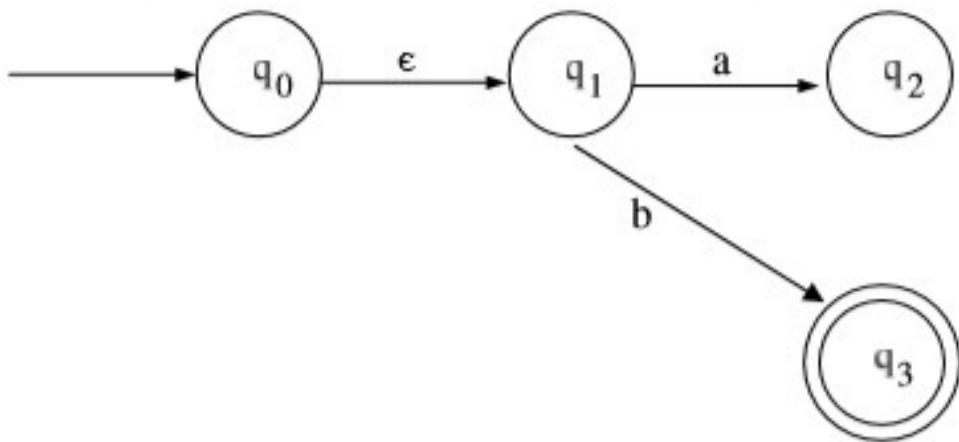
**Tahapan untuk Ekuivalensi NFA dgn  $\epsilon$ \_move dgn DFA dgn  $\epsilon$ \_move:**

- Buat tabel transisi NFA  $\epsilon$ -move dari diagram NFA atau sudah ditentukan semula.
- Carilah  $\epsilon$ -closure untuk setiap state NFA
- Cari setiap fungsi transisi hasil perubahan dari NFA  $\epsilon$ -move ke NFA tanpa  $\epsilon$ -move ( $\delta'$ ) , rumus :

$$\delta'(\text{state}, \text{input}) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(\text{state}, \text{input})))$$

- Berdasarkan langkah sebelumnya, buatlah tabel transisi NFA yg baru tanpa  $\epsilon$ -move
- Tentukan state akhir. Jika State2x pada closure satu state merupakan final state maka state yg baru menjadi final state.

$$F' = F \cup \{q \mid (\epsilon\text{-closure}(q)) \cap F \neq \emptyset\}$$



## Tabel Transisi

$\delta$	a	b
$q_0$	$\emptyset$	$\emptyset$
$q_1$	$\{q_2\}$	$\{q_3\}$
$q_2$	$\emptyset$	$\emptyset$
$q_3$	$\emptyset$	$\emptyset$

$\varepsilon$ -closure dari fsa tersebut

$\varepsilon$ -closure( $q_0$ ) =  $[q_0, q_1]$

$\varepsilon$ -closure( $q_1$ ) =  $[q_1]$

$\varepsilon$ -closure( $q_2$ ) =  $[q_2]$

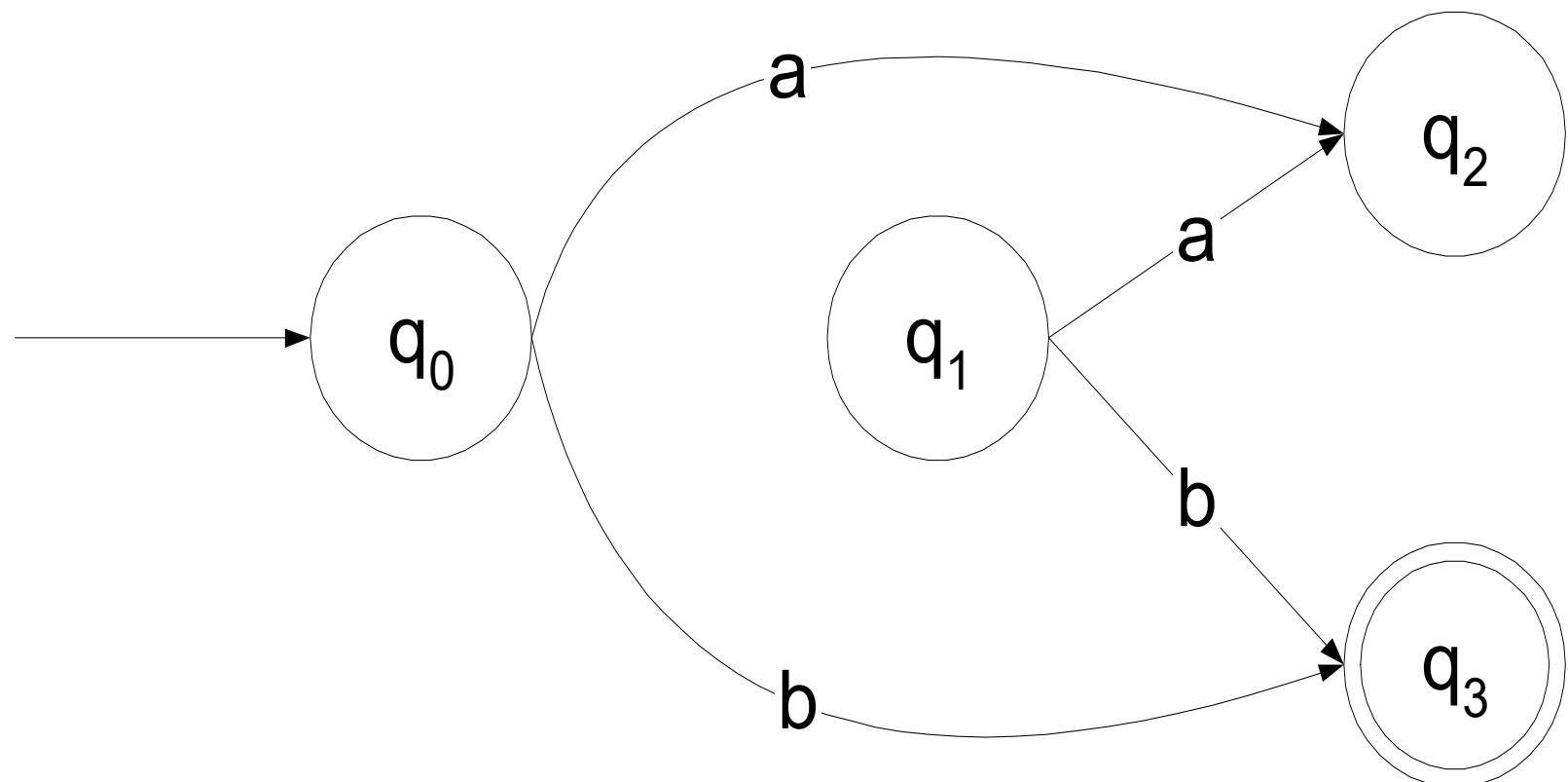
$\varepsilon$ -closure( $q_3$ ) =  $[q_3]$

Cari tabel transisi yang baru ( $\delta'$ ) :

$\delta'$	a	b
$q_0$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_0), a))$ $\varepsilon\text{-cl}(\delta(\{q_0, q_1\}, a))$ $\varepsilon\text{-cl}(q_2)$ $\{q_2\}$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_0), b))$ $\varepsilon\text{-cl}(\delta(\{q_0, q_1\}, b))$ $\varepsilon\text{-cl}(q_3)$ $\{q_3\}$
$q_1$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_1), a))$ $\varepsilon\text{-cl}(\delta(\{q_1\}, a))$ $\varepsilon\text{-cl}(q_2)$ $\{q_2\}$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_1), b))$ $\varepsilon\text{-cl}(\delta(\{q_1\}, b))$ $\varepsilon\text{-cl}(q_3)$ $\{q_3\}$

$\delta'$	a	b
$q_2$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_2), a))$ $\varepsilon\text{-cl}(\delta(\{q_3\}, a))$ $\varepsilon\text{-cl}(\emptyset)$ $\emptyset$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_2), b))$ $\varepsilon\text{-cl}(\delta(\{q_2\}, b))$ $\varepsilon\text{-cl}(\emptyset)$ $\emptyset$
$q_3$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_3), a))$ $\varepsilon\text{-cl}(\delta(\{q_3\}, a))$ $\varepsilon\text{-cl}(\emptyset)$ $\emptyset$	$\varepsilon\text{-cl}(\delta(\varepsilon\text{-cl}(q_3), b))$ $\varepsilon\text{-cl}(\delta(\{q_3\}, b))$ $\varepsilon\text{-cl}(\emptyset)$ $\emptyset$

NFA tanpa  $\epsilon$ -move yang ekuivalen dengan NFA  $\epsilon$ -move



State akhir untuk diagram NFA (gambar 2)

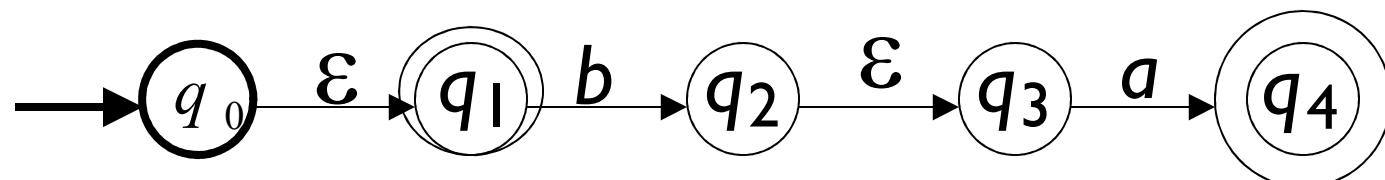
$$F' = F \cup \{q \mid (\text{$\epsilon$-closure}(q) \cap F) \neq \emptyset\}$$

diketahui state akhir / F semula adalah  $\{q_3\}$ .

Cari state lain yang  $\epsilon$ -closure-nya memuat  $q_3$ .

Karena tidak ada maka himpunan state akhir utk NFA gambar adalah tetap  $q_3$

# Ekivalensi NFA dengan $\epsilon$ - Move ke NFA tanpa $\epsilon$ - Move



Gbr . Mesin 5

Penjelasan : dari  $q_2$  tanpa membaca input dapat berpindah ke  $q_3$   
Bagaimana Ekivalensi NFA dengan  $\epsilon$ - Move ke NFA tanpa  $\epsilon$ - Move pada mesin 5?

## Ekuivalensi Non-Deterministic Finite Automata dengan $\epsilon$ -move ke Deterministic Finite Automata tanpa $\epsilon$ -Move

1. Buatlah 5 tuple dari mesin 5

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$

$$S = q_0$$

$$F = \{q_1, q_4\}$$

$\delta$	a	b
q0	$\emptyset$	$\emptyset$
q1	$\Theta$	{q2}
q2	$\emptyset$	$\emptyset$
q3	{q4}	$\emptyset$
q4	$\emptyset$	$\emptyset$

## 2. Membuat $\varepsilon$ -closure untuk setiap state

$\varepsilon$ -Closure ( $q_0$ ) =  $\{q_0, q_1\}$

$\varepsilon$ -Closure ( $q_1$ ) =  $\{q_1\}$

$\varepsilon$ -Closure ( $q_2$ ) =  $\{q_2, q_3\}$

$\varepsilon$ -Closure ( $q_3$ ) =  $\{q_3\}$

$\varepsilon$ -Closure ( $q_4$ ) =  $\{q_4\}$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\delta'(q_0, a) = (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a)))$$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\delta'(q_0, a) = (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a)))$$



( $q_0, q_1$ )

$$\epsilon\text{-Closure}(q_0) = \{q_0, q_1\}$$

$$\epsilon\text{-Closure}(q_1) = \{q_1\}$$

$$\epsilon\text{-Closure}(q_2) = \{q_2, q_3\}$$

$$\epsilon\text{-Closure}(q_3) = \{q_3\}$$

$$\epsilon\text{-Closure}(q_4) = \{q_4\}$$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\delta'(q_0, a) = (\epsilon\text{-Cl}(\delta'(\underline{\epsilon\text{-Cl}(q_0)}, a)))$$

$$(q_0, q_1)$$



$$= (\epsilon\text{-Cl}(\delta'((q_0, q_1), a)))$$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\begin{aligned}\delta'(q_0, a) &= (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a))) \\ &= (\epsilon\text{-Cl}(\delta'((q_0, q_1), a)))\end{aligned}$$

$\delta$	a	b
$q_0$	$\emptyset$	$\emptyset$
$q_1$	$\Theta$	$\{q_2\}$
$q_2$	$\emptyset$	$\emptyset$
$q_3$	$\{q_4\}$	$\emptyset$
$q_4$	$\emptyset$	$\emptyset$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\begin{aligned}\delta'(q_0, a) &= (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a))) \\ &= (\epsilon\text{-Cl}(\delta'((q_0, q_1), a)))\end{aligned}$$



$\theta$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\begin{aligned}\delta'(q_0, a) &= (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a))) \\ &= (\epsilon\text{-Cl}(\delta'((q_0, q_1), a)))\end{aligned}$$



$\theta$

$\delta$	a	b
q0	$\theta$	$\theta$
q1	$\Theta$	{q2}
q2	$\theta$	$\theta$
q3	{q4}	$\theta$
q4	$\theta$	$\theta$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\begin{aligned}\delta'(q_0, a) &= (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a))) \\ &= (\epsilon\text{-Cl}(\delta'((q_0, q_1), a)))\end{aligned}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \theta & U & \theta = \theta \end{array}$$

a.  $\delta'$  state  $q_0$  dengan input a

$$\delta'(q_0, a) = (\varepsilon\text{-Cl}(\delta'(\varepsilon\text{-Cl}(q_0), a)))$$

$$= (\varepsilon\text{-Cl}(\delta'((q_0, q_1), a)))$$

$$\downarrow \quad \downarrow$$

$$\theta \quad U \quad \theta = \theta$$

$$\downarrow$$

$$= (\varepsilon\text{-Cl}(\theta))$$

3. Buatlah transisi ( $\delta'$ ) untuk Ekuivalensi Non-Deterministic Finite Automata dengan  $\epsilon$ -move ke Deterministic Finite Automata tanpa  $\epsilon$ -Move

a.  $\delta'$  state  $q_0$  dengan input a

$$\begin{aligned}\delta'(q_0, a) &= (\epsilon\text{-Cl}(\delta'(\epsilon\text{-Cl}(q_0), a))) \\ &= (\epsilon\text{-Cl}(\delta'(q_0, q_1), a)) \\ &= (\epsilon\text{-Cl}(\theta)) \\ &= \theta\end{aligned}$$

Sehingga  $\delta'(q_0, a) = \theta$

$\delta^*$  untuk setiap state dan input

$\delta^*$	a	b
q0	$\emptyset$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\emptyset$
q3	{q4}	$\emptyset$
q4	$\emptyset$	$\emptyset$

## 4. Membuat mesin NFA tanpa $\epsilon$ - move

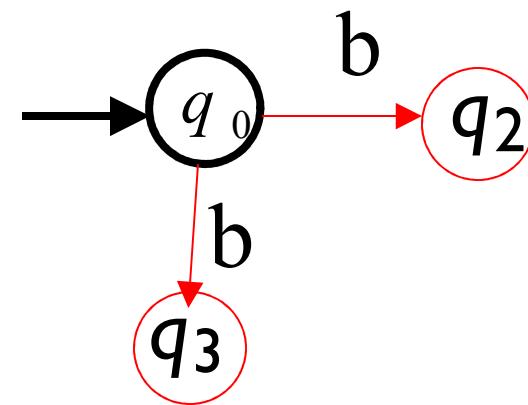
$\delta$ ‘	a	b
q0	$\emptyset$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\emptyset$
q3	{q4}	$\emptyset$
q4	$\emptyset$	$\emptyset$

State q0 input a



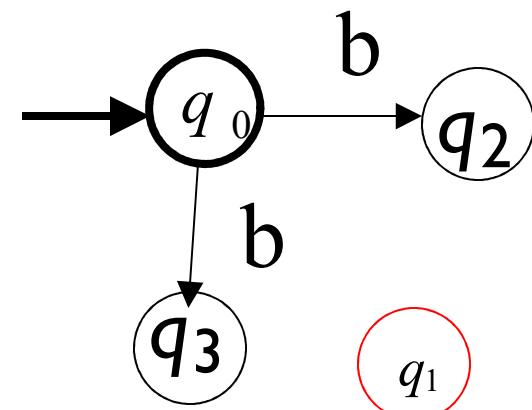
# State q0 input b

$\delta'$	a	b
q0	$\theta$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\theta$
q3	{q4}	$\theta$
q4	$\theta$	$\theta$



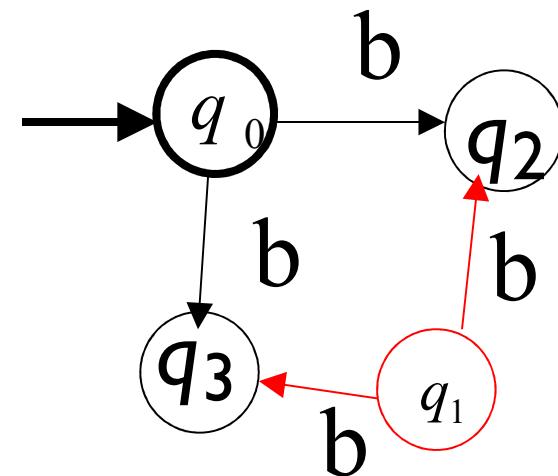
# State q1 input a

$\delta'$	a	b
q0	$\theta$	$\{q2, q3\}$
q1	$\Theta$	$\{q2, q3\}$
q2	$\{q4\}$	$\theta$
q3	$\{q4\}$	$\theta$
q4	$\theta$	$\theta$



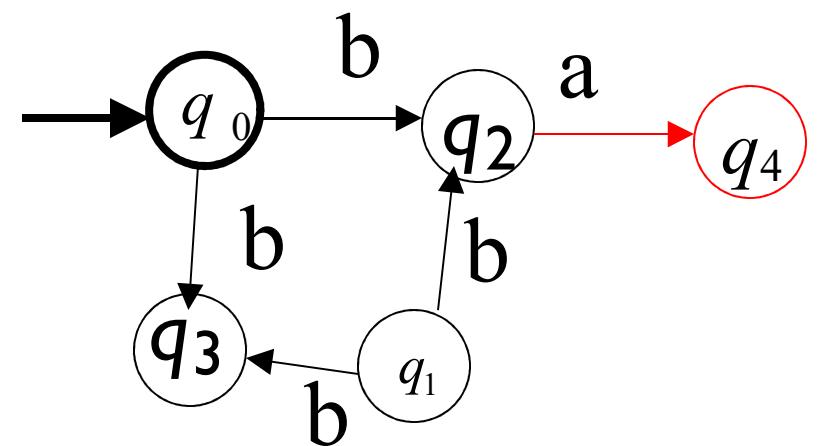
# State q1 input b

$\delta'$	a	b
q0	$\theta$	$\{q2, q3\}$
q1	$\Theta$	$\{q2, q3\}$
q2	$\{q4\}$	$\theta$
q3	$\{q4\}$	$\theta$
q4	$\theta$	$\theta$



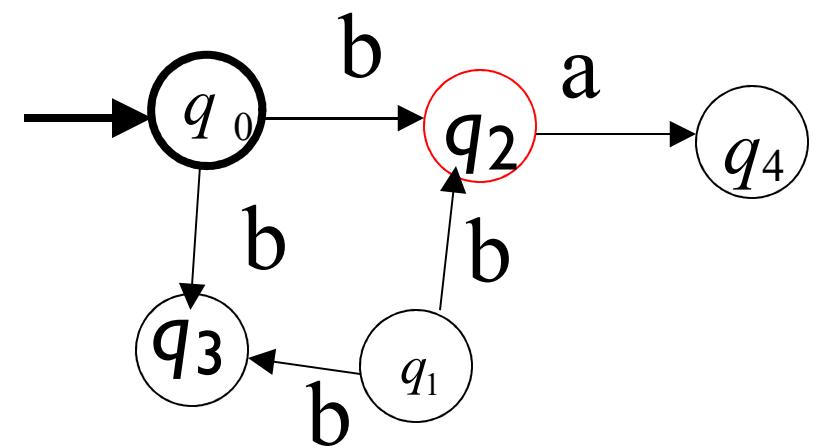
# State q2 input a

$\delta'$	a	b
q0	$\theta$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\theta$
q3	{q4}	$\theta$
q4	$\theta$	$\theta$



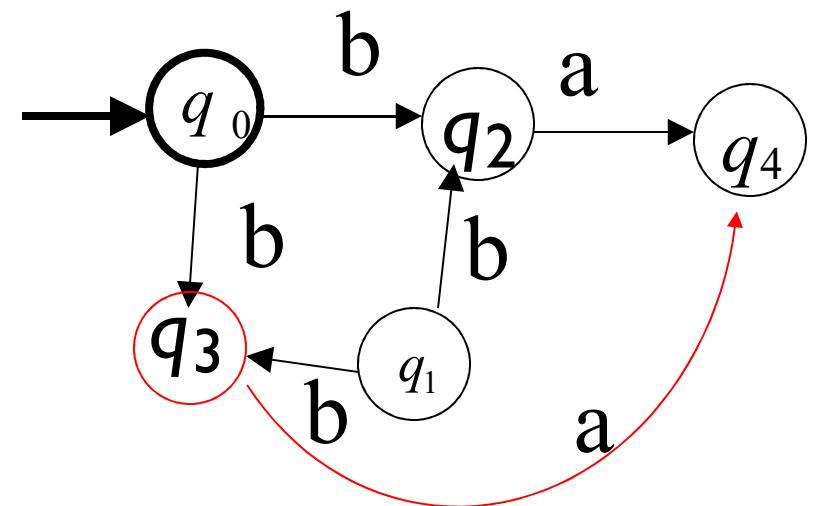
# State q2 input b

$\delta'$	a	b
q0	$\theta$	$\{q2, q3\}$
q1	$\Theta$	$\{q2, q3\}$
q2	$\{q4\}$	$\theta$
q3	$\{q4\}$	$\theta$
q4	$\theta$	$\theta$



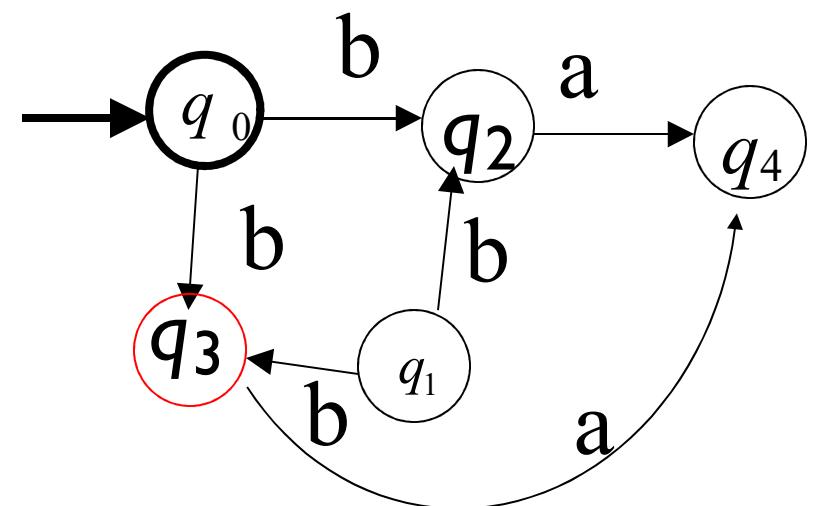
# State q3 input a

$\delta'$	a	b
q0	$\theta$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\theta$
q3	{q4}	$\theta$
q4	$\theta$	$\theta$



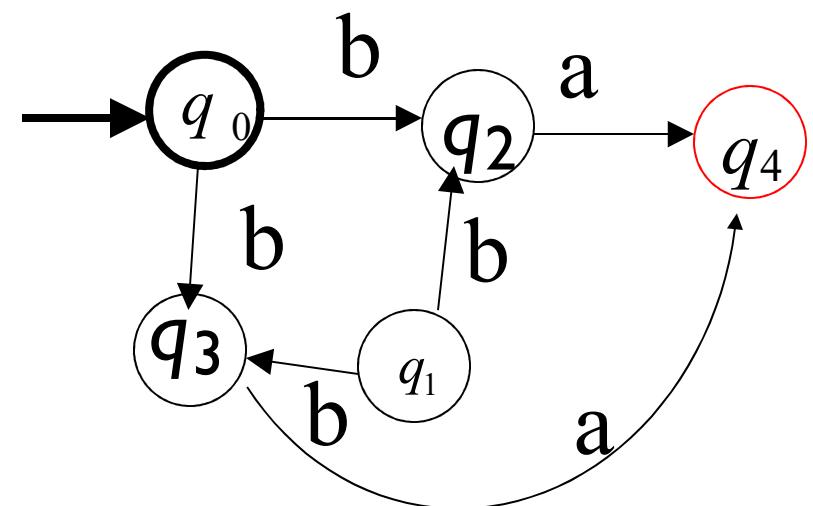
# State q3 input b

$\delta'$	a	b
q0	$\theta$	$\{q2, q3\}$
q1	$\Theta$	$\{q2, q3\}$
q2	$\{q4\}$	$\theta$
q3	$\{q4\}$	$\theta$
q4	$\theta$	$\theta$



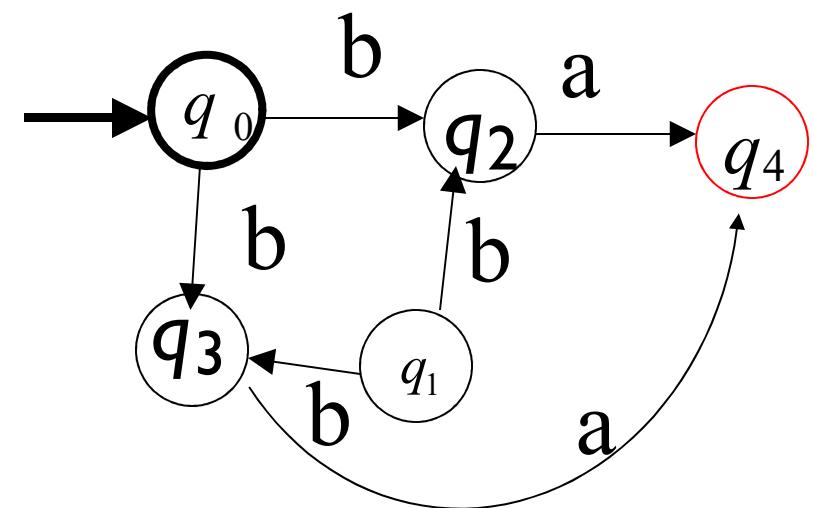
# State q4 input a

$\delta'$	a	b
q0	$\theta$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\theta$
q3	{q4}	$\theta$
q4	$\theta$	$\theta$



# State q4 input b

$\delta'$	a	b
q0	$\theta$	{q2, q3}
q1	$\Theta$	{q2, q3}
q2	{q4}	$\theta$
q3	{q4}	$\theta$
q4	$\theta$	$\theta$



# Menentukan state akhir

1. Tentukan F pada NFA dengan  $\epsilon$ -move

$$F = \{q_1, q_4\}$$

2. Tentukan  $\epsilon$ - closure yang mengandung F pada NFA dengan

$$\epsilon\text{-Closure}(q_0) = \{q_0, q_1\}$$

$$\epsilon\text{-Closure}(q_1) = \{q_1\}$$

$$\epsilon\text{-Closure}(q_2) = \{q_2, q_3\}$$

$$\epsilon\text{-Closure}(q_3) = \{q_3\}$$

$$\epsilon\text{-Closure}(q_4) = \{q_4\}$$

karena

$$\epsilon\text{-Closure}(q_0) = \{q_0, q_1\}$$

$$\epsilon\text{-Closure}(q_1) = \{q_1\}$$

$$\epsilon\text{-Closure}(q_4) = \{q_4\}$$

maka  $F = \{q_0, q_1, q_4\}$

# MESIN5 TANPA $\varepsilon$ -MOVE

