

PERTEMUAN 15

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Penyelesaian Integral Lipat

Penyelesaian integral lipat dua adalah menggunakan teknik pengintegral sebanyak dua kali dimulai dari yang paling dalam dahulu untuk di integralkan.

Rumus

$$\iint_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dx dy$$

Contoh 1

$$\int_0^3 \left[\int_1^2 (2x + 3y) dx \right] dy$$

$$\int_1^2 (2x + 3y) dx = [x^2 + 3yx]_1^2 = 4 + 6y - (1 + 3y) = 3 + 3y$$

$$\int_0^3 \left[\int_1^2 (2x + 3y) dx \right] dy = \int_0^3 [3 + 3y] dy = \left[3y + \frac{3}{2}y^2 \right]_0^3 = 9 + \frac{27}{2} = \frac{45}{2}$$

Contoh 2

$$\begin{aligned} & \int_1^2 \left[\int_0^3 (2x + 3y) dy \right] dx \\ & \int_0^3 \left(2x + 3y \right) dy = \left[2xy + \frac{3}{2}y^2 \right]_0^3 = 6x + \frac{27}{2} \\ & \int_1^2 \left[\int_0^3 (2x + 3y) dy \right] dx = \int_1^2 \left[6x + \frac{27}{2} \right] dx = \left[3x^2 + \frac{27}{2}x \right]_1^2 \\ & = 12 + 27 - \left(3 + \frac{27}{2} \right) \\ & = \frac{45}{2} \end{aligned}$$

Contoh 3

$$\begin{aligned} & \int_0^8 \int_0^4 \frac{1}{16} (64 - 8x + y^2) dx dy \\ & \int_0^4 \frac{1}{16} (64 - 8x + y^2) dx = \frac{1}{16} [64x - 4x^2 + y^2x]_0^4 \\ & = \frac{1}{16} [256 - 64 + 4y^2] \\ & = 12 + \frac{1}{4}y^2 \\ & \int_0^8 \left(12 + \frac{1}{4}y^2 \right) dy = \left[12y + \frac{y^3}{12} \right]_0^8 = 96 + \frac{512}{12} = 96 + \frac{128}{3} = 138\frac{2}{3} \end{aligned}$$

Contoh 4

$$\begin{aligned}\iint_R (x^2 + 2y^2) dA &= \int_0^6 \int_0^4 (x^2 + 2y^2) dy dx \\ &= \int_0^6 \left[x^2 y + \frac{2}{3} y^3 \right]_0^4 dx \\ &= \int_0^6 \left(4x^2 + \frac{128}{3} \right) dx \\ &= \left[\frac{4}{3} x^3 + \frac{128}{3} x \right]_0^6 \\ &= 288 + 256 \\ &= 544\end{aligned}$$

Contoh 5

$$\begin{aligned}\iint_R \sin(x+y) dA &= \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(x+y) dy dx \\ &= \int_0^{\frac{\pi}{2}} [-\cos(x+y)]_0^{\frac{\pi}{2}} dx \\ &= \int_0^{\frac{\pi}{2}} \left(-\cos\left(\frac{\pi}{2} + y\right) + \cos y \right) dx \\ &= [\sin y]_0^{\frac{\pi}{2}} - \left[\sin\left(\frac{\pi}{2} + y\right) \right]_0^{\frac{\pi}{2}} \\ &= \sin\left(\frac{\pi}{2}\right) - \sin(\pi) + \sin\left(\frac{\pi}{2}\right) \\ &= 2\end{aligned}$$

Latihan Soal

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|----|--|-----|---|
| 1. | $\int_0^2 \int_1^3 x^2 y \, dy \, dx$ | 6. | $\int_0^{\frac{\pi}{2}} \int_0^1 x \sin xy \, dy \, dx$ |
| 2. | $\int_{-1}^4 \int_1^2 (x + y^2) \, dy \, dx$ | 7. | $\int_0^1 \int_0^1 x e^{xy} \, dy \, dx$ |
| 3. | $\int_1^2 \int_0^3 (xy + y^2) \, dy \, dx$ | 8. | $\int_0^3 \int_0^1 2x\sqrt{x^2 + y} \, dx \, dy$ |
| 4. | $\int_{-1}^1 \int_1^2 (x^2 + y^2) \, dx \, dy$ | 9. | $\int_0^1 \int_0^1 \frac{y}{(xy + 1)^2} \, dx \, dy$ |
| 5. | $\int_0^{\pi} \int_0^1 x \sin y \, dx \, dy$ | 10. | $\int_0^1 \int_0^2 \frac{y}{1 + x^2} \, dy \, dx$ |