PERTEMUAN 5

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Limit Fungsi Trigonometri

- Rumus dasar Limit Trigonometri

(1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(2) $\lim_{x \to 0} \frac{x}{\sin x} = 1$

(3) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(4) $\lim_{x \to 0} \frac{x}{\tan x} = 1$

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$$\lim_{x\to 0} \frac{\sin ax}{bx} = \lim_{x\to 0} \frac{\sin ax}{bx} \times \frac{a}{a} = \lim_{x\to 0} \frac{\sin ax}{ax} \times \frac{a}{b} = \frac{a}{b}$$

$$\lim_{x\to 0} \frac{bx}{\sin ax} = \lim_{x\to 0} \frac{bx}{\sin ax} \times \frac{a}{a} = \lim_{x\to 0} \frac{ax}{\sin ax} \times \frac{b}{a} = \frac{b}{a}$$

$$\lim_{x\to 0} \frac{\tan ax}{bx} = \lim_{x\to 0} \frac{\tan ax}{bx} \times \frac{a}{a} = \lim_{x\to 0} \frac{\tan ax}{ax} \times \frac{a}{b} = \frac{a}{b}$$

$$\lim_{x\to 0} \frac{bx}{\tan ax} = \lim_{x\to 0} \frac{bx}{\tan ax} \times \frac{a}{a} = \lim_{x\to 0} \frac{ax}{\tan ax} \times \frac{b}{a} = \frac{b}{a}$$

$$\lim_{x\to 0} \frac{\sin ax}{\tan bx} = \lim_{x\to 0} \frac{\sin ax}{\tan bx} \times \frac{ax}{ax} \times \frac{bx}{bx} = \lim_{x\to 0} \frac{\sin ax}{ax} \times \frac{bx}{\tan bx} \times \frac{ax}{bx} = \frac{a}{b}$$

(1) $\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b}$ dan $\lim_{x\to 0} \frac{bx}{\sin ax} = \frac{b}{a}$

(2) $\lim_{x\to 0} \frac{\tan ax}{bx} = \frac{a}{b}$ dan $\lim_{x\to 0} \frac{bx}{\tan ax} = \frac{b}{a}$

(3) $\lim_{x\to 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$ dan $\lim_{x\to 0} \frac{\tan bx}{\sin ax} = \frac{b}{a}$

Contoh Soal

(a) $\lim_{x \to 0} \left[\frac{\sin 4x}{3x} - \frac{\tan 2x}{\sin 6x} + \frac{8x}{\tan 2x} \right] = \frac{4}{3} - \frac{2}{6} + \frac{8}{2} = \frac{4}{3} - \frac{1}{3} + 4 = 5$

(b) $\lim_{x \to 0} \frac{\sin^2 6x}{3x . \tan 4x} = \lim_{x \to 0} \frac{\sin 6x}{3x} \cdot \frac{\sin 6x}{\tan 4x} = \left(\frac{6}{3}\right)\left(\frac{6}{4}\right) = \frac{36}{12} = 3$

(c) $\lim_{x \to 0} \frac{2\tan^2 3x . \sin 2x}{4x^2 . \sin 6x} = \lim_{x \to 0} 2\left(\frac{\tan 3x}{4x}\right)\left(\frac{\tan 3x}{x}\right)\left(\frac{\sin 2x}{\sin 6x}\right)$

$= 2\left(\frac{3}{4}\right)\left(\frac{3}{1}\right)\left(\frac{2}{6}\right)$

$= 3/2$

(d) $\lim_{x \to 0} \frac{6 . \sin^3 2x}{\sin 4x . \sin 3x} = \lim_{x \to 0} 6\left(\frac{\sin 2x}{\sin 4x}\right)\left(\frac{\sin 2x}{\sin 3x}\right)\sin 2x$

$= 6\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\sin 2(0)$

$= \left(\frac{24}{12}\right)0$

$= 0$

(e) $\underset{x\to 0}{\text{Limit}}\left[\frac{\sin 2x+4x}{\sin 3x+\tan x}\right] = \underset{x\to 0}{\text{Limit}}\left[\frac{\frac{\sin 2x}{x}+\frac{4x}{x}}{\frac{\sin 3x}{x}+\frac{\tan x}{x}}\right]$

$= \left[\frac{\frac{2}{1}+\frac{4}{1}}{\frac{3}{1}+\frac{1}{1}}\right]$

$= \left(\frac{2+4}{3+1}\right)$

$= 3/2$

(e) $\underset{x\to 0}{\text{Limit}}\left[\frac{6x^2+\sin^2 3x}{\tan^2 2x-x^2}\right] = \underset{x\to 0}{\text{Limit}}\left[\frac{\frac{6x^2}{x^2}+\frac{\sin^2 3x}{x^2}}{\frac{\tan^2 2x}{x^2}-\frac{x^2}{x^2}}\right]$

$= \left[\frac{\frac{6}{1}+\frac{9}{1}}{\frac{4}{1}-\frac{1}{1}}\right]$

$= \left(\frac{6+9}{4-1}\right)$

$= 5$

Contoh Soal

(a) $\lim_{x \to 2} \frac{3\sin(x-2)}{(4x-8)} = \lim_{x \to 2} \frac{3\sin(x-2)}{4(x-2)} = 3\left(\frac{1}{4}\right) = \frac{3}{4}$

(b) $\lim_{x \to 3} \frac{6\tan^2(2x-6)}{(3x-9)^2} = \lim_{x \to 3} 6\left(\frac{\tan(2x-6)}{3x-9}\right)^2$

$= \lim_{x \to 3} 6\left(\frac{\tan 2(x-3)}{3(x-3)}\right)^2$

$= 6.\left(\frac{2}{3}\right)^2$

$= 8/3$

(c) $\lim_{x \to 4} \left[\frac{\sin(2x-8)}{\tan(x-4)+(3x-12)}\right] = \lim_{x \to 4} \left[\frac{\sin 2(x-4)}{\tan(x-4)+3(x-4)}\right]$

$= \lim_{x \to 4} \left[\frac{\frac{\sin 2(x-4)}{(x-4)}}{\frac{\tan(x-4)}{(x-4)} + \frac{3(x-4)}{(x-4)}}\right]$

$= \left(\frac{2}{1+3}\right)$

$= 1/2$

(d) $$\lim_{x\to 1} \frac{\tan(2x^2-6x+4)}{3x^2-9x+6} = \lim_{x\to 1} \frac{\tan 2(x^2-3x+2)}{3(x^2-3x+2)}$$

$$= \lim_{x\to 1} \frac{\tan 2(x+3)(x-1)}{3(x+3)(x-1)}$$

$$= \frac{2}{3}$$

(e) $$\lim_{x\to 2} \frac{\sin(3x-6)}{x^2+2x-8} = \lim_{x\to 2} \frac{\sin 3(x-2)}{(x+4)(x-2)}$$

$$= \lim_{x\to 2} \frac{1}{(x+4)} \cdot \frac{\sin 3(x-2)}{(x-2)}$$

$$= \left(\frac{1}{2+4}\right)\left(\frac{3}{1}\right)$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Rumus tambahan untuk penyelesaian Limit Trigonometri

(1) $1 - \cos 2\alpha = 2\sin^2\alpha$

(2) $\cos 2\alpha - 1 = -2\sin^2\alpha$

(3) $1 - \cos^2\alpha = \sin^2\alpha$

(4) $\text{ctg}\,\alpha = \dfrac{1}{\tan\alpha}$

(5) $\cos A - \cos B = -2\sin\frac{1}{2}(A + B).\sin\frac{1}{2}(A - B)$

Contoh Soal

(a) $\lim_{x\to 0} \frac{1-\cos 6x}{\cos 8x - 1} = \lim_{x\to 0} \frac{1-\cos 2(3x)}{\cos 2(4x) - 1}$

$= \lim_{x\to 0} \frac{2.\sin^2 3x}{-2.\sin^2 4x}$

$= \lim_{x\to 0} -\left(\frac{.\sin 3x}{.\sin 4x}\right)^2$

$= -\left(\frac{.3}{.4}\right)^2$

$= -\frac{.9}{.16}$

(b) $\lim_{x\to 0} \frac{3\cos 4x - 3}{2.\sin^2 3x} = \lim_{x\to 0} \frac{3(\cos 2(2x) - 1)}{2.\sin^2 3x}$

$= \lim_{x\to 0} \frac{-3.\sin^2 2x}{2.\sin^2 3x}$

$= \lim_{x\to 0} -\frac{3}{2}\left(\frac{.\sin 2x}{.\sin 3x}\right)^2$

$= -\frac{3}{2}\left(\frac{.2}{.3}\right)^2$

$= -\frac{.2}{.3}$

(c) $\underset{x\to 0}{\text{Limit}} \dfrac{4-4\cos 2x}{1-\cos^2 3x} = \underset{x\to 0}{\text{Limit}} \dfrac{4(1-\cos 2x)}{1-\cos^2 3x}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{4.(2\sin^2 x)}{\sin^2 3x}$

$= \underset{x\to 0}{\text{Limit}}\ 8\left(\dfrac{.\sin x}{.\sin 3x}\right)^2$

$= 8\left(\dfrac{1}{.3}\right)^2$

$= \dfrac{8}{9}$

(d) $\underset{x\to 0}{\text{Limit}} \dfrac{2-2\cos^2 6x}{3\cos^2 2x - 3} = \underset{x\to 0}{\text{Limit}} \dfrac{2(1-\cos^2 6x)}{3(\cos^2 2x - 1)}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{4.(\sin^2 6x)}{3.(-\sin^2 2x)}$

$= \underset{x\to 0}{\text{Limit}} -\dfrac{4}{3}\left(\dfrac{\sin 6x}{\sin 2x}\right)^2$

$= -\dfrac{4}{3}\left(\dfrac{6}{2}\right)^2$

$= -\dfrac{4}{3}(9)$

$= -12$

(e) $\underset{x\to 0}{\text{Limit}} \dfrac{\cos 5x - \cos 3x}{1-\cos 8x} = \underset{x\to 0}{\text{Limit}} \dfrac{-2\sin\frac{1}{2}(5x+3x).\sin\frac{1}{2}(5x-3x)}{1-\cos 2(4x)}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{-2\sin 4x.\sin x}{2\sin^2 4x}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{-\sin x}{\sin 4x}$

$= -\dfrac{1}{4}$

(f) $\underset{x\to 0}{\text{Limit}} \dfrac{3\cos 6x - 3\cos 2x}{1-\cos^2 2x} = \underset{x\to 0}{\text{Limit}} \dfrac{3(\cos 6x - \cos 2x)}{1-\cos^2 2x}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{-6\sin\frac{1}{2}(6x+2x).\sin\frac{1}{2}(6x-2x)}{\sin^2(2x)}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{-6\sin 4x.\sin 2x}{\sin 2x.\sin 2x}$

$= \underset{x\to 0}{\text{Limit}} \dfrac{-6\sin 4x}{\sin 2x}$

$= -6\left(\dfrac{4}{2}\right)$

$= -12$

Contoh Soal Lainya

(a) $\underset{x\to\frac{\pi}{2}}{\text{Limit}} \dfrac{3.\sin 4x}{\cos x}$

(b) $\underset{x\to\frac{\pi}{4}}{\text{Limit}} \dfrac{\sin 6x + \sin 2x}{\cos 5x + \cos x}$

Jawab

(a) $\underset{x\to\frac{\pi}{2}}{\text{Limit}} \dfrac{3.\sin 4x}{\cos x} = \underset{x\to\frac{\pi}{2}}{\text{Limit}} \dfrac{3.(2\sin 2x.\cos 2x)}{\cos x}$

$= \underset{x\to\frac{\pi}{2}}{\text{Limit}} \dfrac{6\sin 2x.\cos 2x}{\cos x}$

$= \underset{x\to\frac{\pi}{2}}{\text{Limit}} \dfrac{6.(2\sin x.\cos x).\cos 2x}{\cos x}$

$= \underset{x\to\frac{\pi}{2}}{\text{Limit}}$ 12.sin x . cos 2x

$$= 12 \sin\frac{\pi}{2}.\cos 2\left(\frac{\pi}{2}\right)$$

$$= 12 \sin\frac{\pi}{2}.\cos \pi$$

$$= 12\,(1)(0)$$

$$= 0$$

(b) $$\underset{x\to\frac{\pi}{4}}{\text{Limit}} \frac{\sin 6x + \sin 2x}{\cos 5x + \cos x} = \underset{x\to\frac{\pi}{4}}{\text{Limit}} \frac{2.\sin 3x.\cos 2x}{2.\cos 4x.\cos 2x}$$

$$= \underset{x\to\frac{\pi}{4}}{\text{Limit}} \frac{\sin 3x}{\cos 4x}$$

$$= \frac{\sin 3\left(\frac{\pi}{4}\right)}{\cos 4\left(\frac{\pi}{4}\right)}$$

$$= \frac{\sin \frac{3\pi}{4}}{\cos \pi}$$

$$= \frac{\frac{1}{2}\sqrt{3}}{-1}$$

$$= -\frac{1}{2}\sqrt{3}$$

Latihan Soal

$\lim_{x\to 2} \frac{1-\cos^2(x-2)}{3x^2-12x+12} = \ldots$

$\lim_{x\to 0} \frac{\cos 4x - 1}{x \tan 2x} = \ldots$

$\lim_{x\to \frac{\pi}{4}} \frac{1-2\sin^2 x}{\cos x - \sin x} = \ldots$

$\lim_{x\to 0} \frac{(x^2-1)\sin 6x}{x^3+3x^2+2x} = \ldots$

$\lim_{x\to \pi} \frac{x^2 \sin 2x}{x-\pi} = \ldots$

$\lim_{x\to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \ldots$

$\lim_{x\to \frac{\pi}{4}} \frac{1-\sin 2x}{\cos^2 2x} = \ldots$

Nilai $\lim_{x\to 0} \frac{1-\cos 2x}{4x^2} = \ldots$

$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \ldots$

$\lim_{x\to 1} \left(\frac{x^3-1}{x-1} + \frac{\cos(\frac{\pi}{2} - x + 1}{x-1} \right) = \ldots$

$\lim_{x\to 0} \frac{\sin \frac{1}{2}x \tan 2\sqrt{x}}{x\sqrt{x}} = \ldots$

$\lim_{x\to 2} \frac{(t^2-5t+6)\sin(t-2)}{(t^2-t-2)^2} = \ldots$

$\lim_{x\to a} \frac{x-a}{3x-3a+\tan(x-a)} = \ldots$

$\lim_{x\to 0} \frac{\sin 3x - \sin 3x \cos 2x}{2x^2}$

$\lim_{x\to 0} \frac{\sin 6x}{\sin 2x} = \ldots$

$\lim_{h\to 0} \frac{\sin(\frac{1}{3}\pi + h) - \sin \frac{1}{3}\pi}{h} = \ldots$

$\lim_{x\to 0} \frac{\tan x}{x^2+2x} = \ldots$

$\lim_{x\to 4} \frac{\sin(4-2\sqrt{x})}{4-x}$

$\lim_{x\to 0} \frac{2x \sin 3x}{1-\cos 6x} = \ldots$

$\lim_{x\to 0} \frac{\cos x - \cos 2x}{x^2} = \ldots$

$\lim_{x\to \infty} x^2 \sin \frac{1}{x} \tan \frac{1}{x} = \ldots$